Computational modeling and oscillations damping of axial vibrations in a drilling system

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***Abstract*—Drilling operations face with three types of vibrations – torsional, axial and lateral, which are coupled oscillations that occur from the interaction of the drillstring with the rock at the bottom of the borehole. The paper focuses on the numerical and computational modeling of the axial vibrations of a drilling system, considering the case of distributed parameters as being tight connected with the physical description of these phenomena and providing high accuracy of system behavior. The system is modeled by hyperbolic partial differential equations with control and perturbation input signals. The computational solution is derived by means of a well-structured procedure which relies on a "convergent *Method of Lines*" – a semianalytical method used to transform the infinite dimensional system in a finite dimensional one. The computational issues are then tackled via the paradigm of cell-based neural networks which exploit the peculiarities of the approximate system previous obtained. The numerical simulations allow a better understanding of the system dynamics regarding its transient and long-term behavior.**

***Index Terms*—Distributed parameter systems, hyperbolic partial differential equations, computational modeling, drillstring axial vibrations, drillstring**

I. Introduction

The complex coupled oscillations which occur in drilling operations are the main cause of loss of performance and equipment failure in oil extraction industry, leading to permanent costs but also to permanent research efforts for modeling, analyzing and control such undesirable phenomena.

Drilling operations face with three types of vibrations – torsional, axial and lateral – coupled oscillations that occur from the interaction of the drillstring with the rock at the bottom of the borehole [1]. More specifically, the undesirable phenomena that occur in the drilling process may be classified as follows:

- vibrations – phenomena occurring along the drillstring, as combined torsional, axial and lateral oscillations, each of them having a specific signature in equipment depreciation;
- phenomena occurring from the interaction of the bit with the rock:
  - the stick-slip phenomenon due to the rotation motion, with a drastic variation of the angular velocity of the bit (connected with the torsional vibrations),
  - the bit-bounce phenomenon due to the longitudinal (axial) motion, with a drastic variation of the weight on the bit at the bit-rock interface (connected with the axial vibrations),
  - bending motion due to the out-of-balance of the drillstring (connected with the lateral vibrations).

The vibrations of drilling plants are studied as coupled oscillations or as cascaded effect of these dynamics, the most analyzed models being those of the combined torsional and axial vibrations, but also those of decoupled torsional vibrations. These phenomena can be studied on different types of mathematical models, such as lumped parameter models, distributed parameter models, functional differential equations of neutral type as well as coupled PDEs-ODEs models [2], [3], [4], [5].

In this paper we focus on the model of axial vibrations for a drilling plant subjected to control and perturbation input signals. More specifically, we address here the issue of an accurate description of the physical phenomenon as well as of the system behavior by means of its computational model derived for the mathematical model with distributed parameters. Worth mentioning here that, since axial dynamics is driven by the torsional one, it is less studied as a decoupled dynamics, especially within the framework of distributed parameter systems (DPS).

For systems with distributed parameters arisen from different engineering applications and industry processes, the complexity of the mathematical models does not allow obtaining the analytical solution which enables a better understanding of the underlying behavior. In such cases, the solution derived through numerical approaches can give an insight on the real phenomena only if the methods and procedures used for obtaining the computational solution are such that they ensure convergence as well as the preservation of the basic properties of the "true" solution, but also of its Lyapunov stability. On the other hand, the computational implementations of systems with distributed parameters can lead to undesirable results due to numerical instability and systematic errors, but also to computation difficulties related to time and resources consuming due to a great computing effort [6].

In order to cope with these issues, we shall use here the

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computational procedure introduced in [7] by the first author of this paper. This procedure employs the regularity and sparsity of the approximate system, obtained *via* a specific approach of the *Method of Lines*" (MoL), in order to perform computation by means of some well-structured devices belonging to *Artificial Intelligence* field.

The rest of this contribution is organized as follows: Section 2 introduces the mathematical model for the axial vibrations of a drilling mechanism and state the problem; Section 3 concerns the numerical and computational modeling; Section 4 discuss the results of simulations; some conclusions are presented in the last section.

II. The mathematical model and problem formulation

From the mathematical modeling point of view, the important components of a drilling process are: (1) the drillstring, which consists of drill pipe and bottom hole assembly (BHA) and (2) the drilling fluid (the so-called mud). While the drilling mud is responsible for bit cooling and lubricating as well as for ensuring the necessary hydrostatic pressure and maintaining the wellbore stability, the penetration of the rock is the combined result of a rotation motion of the drillstring controlled by the rotary table motor at the top end of the wellbore and of a longitudinal translation motion towards the bottom end of the wellbore due to the BHA which provides force to the bit in order to break the rock.

In this paper, we consider the distributed parameter model for the axial vibrations of a drillstring viewed as a flexible rod, the case of negligible distributed damping. The usual infinite dimensional model for reproducing the oscillatory dynamics is the wave equation. By denoting $z(s,t)$ – the longitudinal displacement of a drillstring of length L, the mathematical model is a second-order one-dimensional (1D) hyperbolic partial differential equation (hPDE) of the form [4], [8]:

$$\frac{\partial^2 z}{\partial t^2}(s,t) = \alpha^2 \frac{\partial^2 z}{\partial s^2}(s,t), \;\; t \geq 0, \;\; 0 < s < L \tag{1}$$

with the boundary conditions (BCs)

$$\begin{aligned} E\Gamma \frac{\partial z}{\partial s}(0,t) &= \beta \frac{\partial z}{\partial t}(0,t) - f_a(t) \\ E\Gamma \frac{\partial z}{\partial s}(L,t) &= -m_b \frac{\partial^2 z}{\partial t^2}(L,t) - f_p(t) \end{aligned} \tag{2}$$

where $s = 0$ corresponds to the top end of the drillstring, while $s = L$ refers to the bottom end of the drillstring.

The notations for parameters are: E – Young modulus, Γ – cross-section of the drillstring, $\alpha = \sqrt{\frac{E}{\rho_a}}$ with ρ_a – mass density of the string, β – viscous friction coefficient, m_b – BHA mass. In the case of decoupled axial dynamics, the forces acting on the boundaries of the drilling system are as follows:

- the boundary $s = 0$:
  - the active force, which is a penetration force

$$f_a(t) = \beta(u_a(t) - \bar{v}_a) \tag{3}$$

with:

- $u_a(t)$ – the stabilizing controller for axial vibration damping and eliminating the bit-bounce phenomenon
- $\bar{v}_a$ – the reference axial velocity;

- the boundary $s = L$:
  - the perturbation force, which is a torsional frictional force that occurs at the bit-rock interface; it is a function of the angular velocity of the bit $\dot{\theta}_b(t) := \frac{\partial \theta}{\partial t}(L,t)$, i.e.

$$f_p(t) = \mu T(\dot{\theta}_b(t)) \tag{4}$$

with:

- μ – a parameter which depends on the friction coefficient at the bit-rock interface and on the bit geometry [1], [9]
- $T(\cdot)$ the bit-rock frictional torque of the form [10]:

$$T(\dot{\theta}_b(t)) = \frac{2pk\dot{\theta}_b(t)}{\dot{\theta}_b(t)^2 + k^2}, \;\; p > 0, \;\; k > 0 \tag{5}$$

where, p – friction force amplitude and k – constant of the friction top angle.

For improving the numerical conditioning, we write system (1)–(2) with respect to the normalized length of the drillstring $x = s/L$

$$\frac{\partial^2 z}{\partial t^2}(x,t) - \frac{\alpha^2}{L^2}\frac{\partial^2 z}{\partial x^2}(x,t) = 0, \;\; t \geq 0, \;\; 0 < x < 1 \tag{6}$$

$$\begin{aligned} \frac{E\Gamma}{L}\frac{\partial z}{\partial x}(0,t) &= \beta \frac{\partial z}{\partial t}(0,t) - f_a(t) \\ \frac{E\Gamma}{L}\frac{\partial z}{\partial x}(1,t) &= -m_b \frac{\partial^2 z}{\partial t^2}(1,t) - f_p(t). \end{aligned} \tag{7}$$

Consider the system of axial vibrations (6)–(7). In order to have a good representation of the system behavior, the aim of this paper is to obtain accurate numerical and computational models, i.e., models that ensure the preservation of the basic properties (existence, uniqueness and data dependence) of the "true" solution, and also the preservation of the Lyapunov stability of this solution.

III. Numerical and computational modeling

In the sequel we shall make use of the computational procedure introduced in [7]. As a result which lies at the interface between the fields of *Computational Mathematics* and *Artificial Intelligence*, this procedure relies on a "convergent *Method of Lines*" for performing numerics and on the paradigm of cell-based dynamical neural networks for computational implementation. Mentioning that the *Method of Lines* is more a guide for solving PDEs than a specific procedure, we emphasize that the approach of MoL we use in our procedure ensures from the beginning the aforementioned requirements of the *Numerical Mathematics* field point of view [11] as well as that of the augmented validation [12]

which additionally includes the requirement for fulfilling the "inherent" stability of the Cetaev's postulate.

From the point of view of *Neural Mathematics* – a new branch of *Computational Mathematics*, the procedure is devoted to the formalized problem solution for the class of (possibly nonlinear) distributed parameter systems (DPS) modeled by second-order 1D hPDE with "nonstandard boundary conditions" which may include: systems of ODEs, derivative BCs and even BCs described by some nonlinear Voltera operators [11].

Consider the system for the drillstring axial vibrations (6)–(7). We introduce the distributed variables $v(x,t) = \frac{\partial z}{\partial t}(x,t)$ and $w(x,t) = \frac{\partial z}{\partial x}(x,t)$ in order to obtain for the hPDE (6) the symmetric Friedrichs form which reads as

$$\begin{aligned} &\frac{\partial v}{\partial t}(x,t) - \frac{\alpha^2}{L^2}\frac{\partial w}{\partial x}(x,t) = 0,\ t \geq 0,\ 0 < x < 1 \\ &\frac{\partial w}{\partial t}(x,t) - \frac{\partial v}{\partial x}(x,t) = 0 \end{aligned} \tag{8}$$

This form further leads, *via* diagonalization, to the normal hyperbolic form of the Riemann invariants, used here for decoupling the system equations. The boundary conditions (7) written with respect to the new variables read as

$$\begin{aligned} &\frac{E\Gamma}{L}w(0,t) = \beta v(0,t) - f_a(t) \\ &\frac{E\Gamma}{L}w(1,t) = -m_b\frac{\partial v}{\partial t}(1,t) - f_p(t). \end{aligned} \tag{9}$$

To equations (8)–(9) we add the initial conditions (ICs)

$$v(x,0) = v_0(x),\ w(x,0) = w_0(x),\ x \in [0,1] \tag{10}$$

and finally obtain a mixed initial-boundary value problem for hyperbolic PDEs.

Now, by taking into account the eigenvalues $\pm\lambda = \pm\alpha/L$ of the system matrix in (8), we derive the PDEs in the normal form of the Riemann invariants $r^+(x,t)$ and $r^-(x,t)$ (see *Section 3.3* in [7]) defined by

$$\begin{cases} r^+(x,t) = v(x,t) + \lambda w(x,t) \\ \\ r^-(x,t) = v(x,t) - \lambda w(x,t) \end{cases} \tag{11}$$

with the converse representation

$$\begin{cases} v(x,t) = \frac{1}{2}(r^+(x,t) + r^-(x,t)) \\ \\ w(x,t) = -\frac{1}{2\lambda}(r^+(x,t) - r^-(x,t)). \end{cases} \tag{12}$$

As already mentioned, the system of hyperbolic PDEs with respect to the Riemann invariants is completely decoupled; it reads as

$$\begin{aligned} &\frac{\partial r^+}{\partial t}(x,t) + \lambda\frac{\partial r^+}{\partial x}(x,t) = 0,\ t \geq 0,\ 0 < x < 1 \\ &\frac{\partial r^-}{\partial t}(x,t) - \lambda\frac{\partial r^-}{\partial x}(x,t) = 0 \end{aligned} \tag{13}$$

where we denoted $r^+(x,t)$ the forward wave and $r^-(x,t)$ the backward wave. From the BCs (9) we obtain the expressions for the top and bottom ends of the drillstring, expressions which link the two waves $r^{\pm}(x,t)$ and which will be further used in the approximate system (15) for their corresponding state variables

$$\begin{aligned} &r^+(0,t) = br^-(0,t) + c(u_a(t) - \bar{v}_a) \\ &\frac{\partial r^-}{\partial t}(1,t) = -\frac{\partial r^+}{\partial t}(1,t) + er^+(1,t) - er^-(1,t) - gf_p(t) \end{aligned} \tag{14}$$

where the notations are

$$a = \Gamma\sqrt{E\rho_a},\ b = \frac{a-\beta}{a+\beta},\ c = \frac{2\beta}{a+\beta},\ e = \frac{a}{m_b},\ g = \frac{2}{m_b}.$$

Following the line of [7], we apply to system (13)–(14) the specific approach for a convergent MoL. Thus, we consider the discretization $h = 1/N$ of $[0,1]$ – the interval of variation of the independent space variable x and then use in (13) the Courant-Isaacson-Rees rule for approximating the derivatives of the Riemann invariants with respect to x. Next, we introduce the approximate functions $\sigma_i^{\pm}(t) \approx r^{\pm}(x_i,t)$, $i = \overline{0,N}$ and, taking into account the BCs (14), we finally obtain an initial value problem for a system of ODEs which approximates the initial-boundary value problem for the system of hyperbolic PDEs (8)–(9)–(10). To conclude, in this manner we associated to the infinite dimensional system (1)–(2) the finite dimensional system which reads as follows

$$\begin{cases} \dot{\sigma}_1^+(t) = bd\sigma_0^- - d\sigma_1^+ - cdu_a(t) + cd\bar{v}_a \\ \\ \dot{\sigma}_i^+(t) = -d\sigma_i^+ + d\sigma_{i-1}^+,\ i = \overline{2,N} \\ \\ \dot{\sigma}_i^-(t) = d\sigma_{i+1}^- - d\sigma_i^-,\ i = \overline{0,N-1} \\ \\ \dot{\sigma}_N^- = -d\sigma_{N-1}^+ + (d+e)\sigma_N^+ - e\sigma_N^- - gf_p(t) \end{cases} \tag{15}$$

where $d = \alpha N/L$.

Worth mentioning here that, from the mathematical point of view, the correct application of these steps of the procedure ensures the good properties for the approximate system concerning the convergence of the approximate solution to the "true" (or analytical) one as well as the preservation of the basic properties of this "true" solution including its Lyapunov stability.

In order to reduce the rounding errors, the number of numerical integrations and thus the computational time, the above system will be implemented by means of the paradigm of cell-based neural networks – nonlinear dynamical systems build up of a (possibly) huge number of cells. The CNNs are well-structured computational structures which can be casted in different topologies such as one-, two- or three-dimensional arrays of cells – to give a few examples. These structures, belonging to the *Artificial Intelligence* field, allow different implementations such as software, combined hardware-software

as well as hardware implementations. We shall use here the software emulation of such structures.

The elementary computing unit – the cell – is characterized by local interconnections with the neighboring cells, different types of inputs and a piecewise-linear activation function. More precisely, in the case of *Cellular Neural Networks* (CNNs), the canonical equation for a cell i within an one-dimensional array of cells of dimension M, reads as [13] (see also [6], [14]):

$$\dot{x}_i(t) = \sum_{j\in N_r(i)} T_{ij}^A f(x_j) + \sum_{j\in N_r(i)} T_{ij}^B u_j + \sum_{j\in N_r(i)} T_{ij}^C x_j + I_i \quad (16)$$

with the notations [13]: x_i – the i^{th} cell state variable, $N_r(i)$ – the r-neighborhood of interactions for the i^{th} cell, u_j – the control variable from $N_r(i)$, T_{ij}^A – an element of the feedback cloning template $\mathbf{T}^A$, T_{ij}^B – an element of the control cloning template $\mathbf{T}^B$, T_{ij}^C – an element of the state feedback cloning template $\mathbf{T}^C$, I_i – a bias or an external stimulus. The nonlinear function $f:\mathbb{R}\to\mathbb{R}$ is the unit bipolar ramp function described by [13]:

$$f(x) = \frac{1}{2}(|x+1| - |x-1|), \quad (17)$$

i.e., a continuous, bounded, nondecreasing, piecewise-linear and globally Lipschitzian function, with the Lipschitz constant $L=1$, verifying also [6]

$$0 < \frac{f(x)}{x} \le L, \;\; f(0) = 0. \quad (18)$$

In view of the above ideas and tools, in order to cast system (15) in a cell-based structure and to yield the "cloning templates", a flow diagram suggests the rearrangement given by the following state vector $y\in\mathbb{R}^{2N+1}$

$$y = [\sigma_1^+ \;\; \sigma_2^+ \;\; \ldots \;\; \sigma_N^+ \;\; \sigma_N^- \;\; \sigma_{N-1}^- \;\; \ldots \;\; \sigma_1^- \;\; \sigma_0^-]^T. \quad (19)$$

Consequently, by comparison of these equations with the canonical equation (16), we obtain the description of the software emulated CNN used for implementation. Thus, the state cloning template

$$T^C = [0 \;\; d \;\; -d] \quad (20)$$

is used for computing the dynamics of $2N-1$ inner cells $i = \overline{3,N}\cup\overline{N+2,2N+1}$, i.e.,

$$\dot{y}_i = [0 \;\; d \;\; -d] \begin{bmatrix} y_{i-2} \\ y_{i-1} \\ y_i \end{bmatrix}. \quad (21)$$

In this case, only three state templates are different from (20), those written for the corner cells of the interconnection (system) matrix. The cells dynamics is without inputs from the neighboring cells, thus $T^B = 0$. Also, we can identify the bias $I_{11} = -cd\bar{v}_a$ and two external stimuli $I_{12} = cdu_a(t)$ and $I_N = -2gf_p(t)$.

The main advantage introduced by the underlying "philosophy" of the *Method of Lines* relies on the existence of many and specific high performance commercial ODEs solvers used for numerical integration with respect to the time independent variable. Moreover, this fact ensures a finer discretization with respect to the time variable than that for the space variable, leading to a low number of numerical integrations. Having at our disposal the approximate solutions $\sigma_i^{\pm}(t)$, $i=\overline{0,N}$ for the Riemann invariants $r^{\pm}(x,t)$, $x\in[0,1]$, $t\ge 0$, we can then reconstruct the space-time evolution of the distributed variables $v(x,t)$ and $w(x,t)$ by using the inverse transformation (12) – more precisely, we obtain the approximate functions $\nu_i(t)\approx v(x_i,t)$ and $\omega_i(t)\approx w(x_i,t)$ for $i=\overline{0,N}$.

The efficiency of the procedure can be also discussed from the point of view of the computational issues. We can conclude that in this case, the number of numerical function evaluations is only 4, and this is true regardless of the magnitude of discretization points N. Consequently, this fact leads to reduced computational effort and, thus, computational time as well as to reduced cumulated rounding errors.

IV. Simulation results

Considering the finite-dimensional approximate system (15), several simulations were performed by using the software environment, MATLAB. In order to cope with the stiffness issues and to exploit the sparsity of the interconnection matrix, the numerical integrations were performed by using the ode15s solver for $N=10$ discretization points of the interval $[0,1]$.

The simulation values for the system parameters are as follows [1]: $L = 1172$ m, $E = 200\times 10^9$ N m^{-2}, $\Gamma = 35\times 10^{-4}$ m^2, $m_b = 37278$ kg, $\rho_a = 2000$ N m s, $\beta = 200.025$ $kg\ s^{-1}$. Also, we assumed the system is subjected to constant angular and axial velocities, i.e. $\bar{\omega} = 10$ rad s^{-1} and $\bar{v}_a = 0.1$ m s^{-1}. For the forces acting at the two ends of the drillstring, the following assumptions were made:

- The active force f_a given in (3) with the control $|u_a|\le \bar{u_a}$, as a function of the axial displacement of the bit
$$u_a(t) = K_a w(1,t) = -\frac{LK_a}{2\alpha}(\sigma_N^+ - \sigma_N^-) \quad (22)$$
with the proportional factor $K_a = 235$.
- The perturbation force f_p is a slow damped signal with the shape depicted in Fig. 1 (in accordance with the results and descriptions in [1], [3]) and described by (4) with $\mu = 257$ m^{-1}, $p = 300$ and $k = 0.3$; in order to mimic the oscillatory behavior of angular velocity of the bit (torsional vibrations), we used here
$$\dot{\theta}_b(t) = 0.2\frac{e^{-\xi\bar{\omega}t}}{\sqrt{1-\xi^2}} sin\left(\bar{\omega}t\sqrt{1-\xi^2}\right) \quad (23)$$
with $\xi = 0.03$.

Analyzing the graphical representations presented in this section, firstly one can easily observe the initial oscillatory behavior for both distributed variables, axial velocity $v(x,t)$ and axial deviation $w(x,t)$, as a consequence of the perturbation $f_p(t)$ due to the simulated torsional vibrations at the bit level. Figs. 2 and 3 show the oscillatory behavior at the level of the bit-rock contact surface. Moreover, the bit-bonce

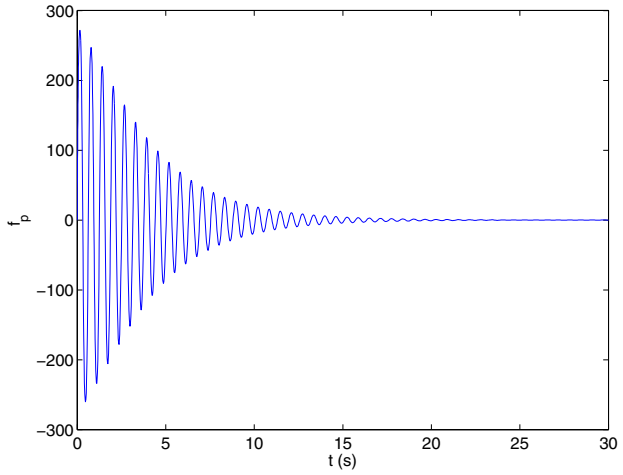

Fig. 1. The perturbation function $f_p(t)$ given in (4).

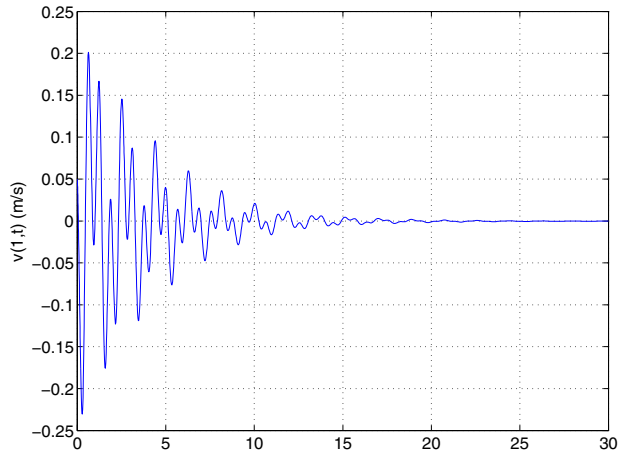

Fig. 2. The axial velocity at the bit level.

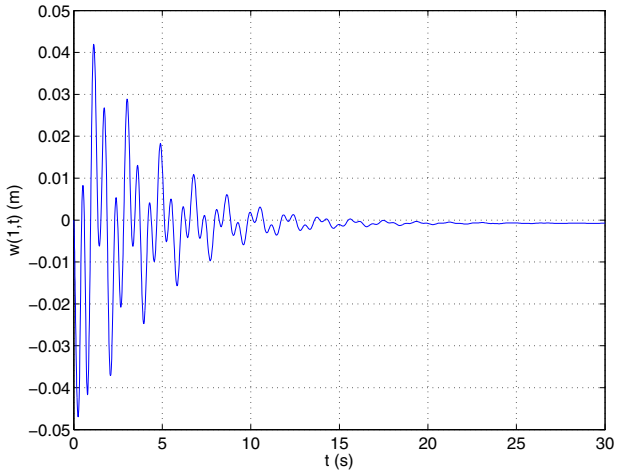

Fig. 3. The axial deviation at the bit level.

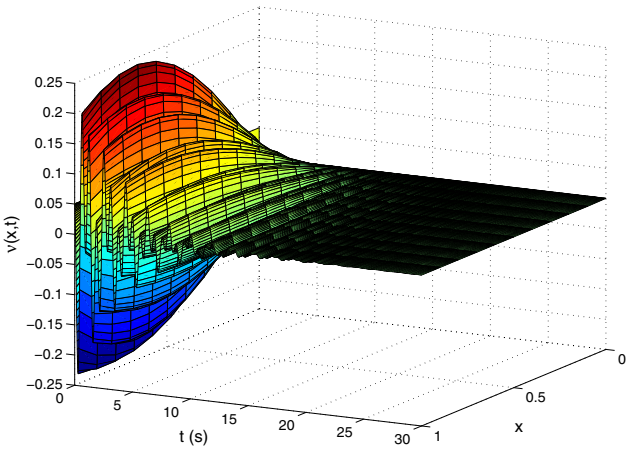

Fig. 4. 3D representation for the approximate solution $v_i(t)$, $i = \overline{0,10}$ of the distributed variable $v(x,t)$, $x \in [0,1]$, i.e., the axial velocity along the normalized length of the drillstring.

phenomenon, due to the repetitive loss of contact between the bit and the rock, is well emphasized in these two plots.

On the other hand, the 3D representations of the axial velocity and the axial displacement, presented in Figs. 4 and 5, provide information regarding the transient as well as the global behavior of the axial vibrations system – a useful information for evaluating the physical phenomenon, the vibrations amplitude and their distributions along the normalized length of the drillstring.

V. Conclusions

The paper considers the numerical and computational modeling of the axial vibrations encountered in drilling operations from the oil extraction industry. Mentioning that these processes are the main cause of equipment failure and permanent costs, the usefulness of such computational procedures as well as of the computational solutions for a better understanding the underlying phenomena is obvious.

The drillstring vibrations can be described by different mathematical models: lumped parameters model, distributed parameters model, neutral-type time-delay model or combined ODEs-PDEs model [4]. The computational procedure we used here allows the analysis of such phenomena by considering the system's model with distributed parameters – described by hPDEs with nonstandard BCs (see, for instance, [7], [11], [15] and the references herein) – i.e. the mathematical model most connected with the physical phenomenon.

From the last section we can see that the simulation results are in accordance with the physical phenomena under consideration. They give an insight regarding the effectiveness of the computational procedure employed and its usefulness in providing information tight related with the real phenomena.

To conclude, considering the system of drillstring axial vibrations described by the infinite dimensional system (1)–

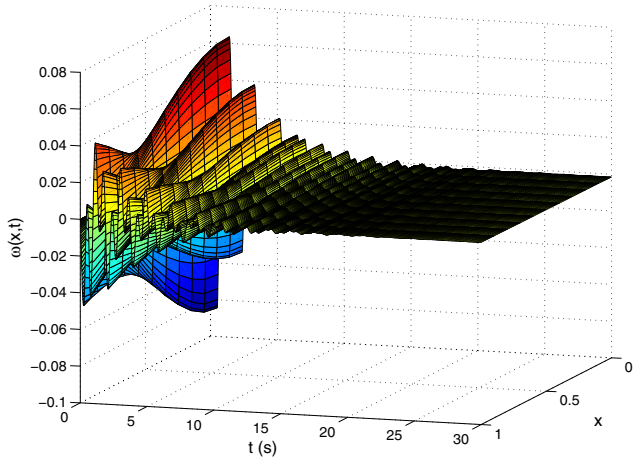

Fig. 5. 3D representation for the approximate solution $\omega_i(t)$, $i=\overline{0,10}$ of the distributed variable $w(x,t)$, $x\in[0,1]$, i.e., the axial displacement along the normalized length of the drillstring.

(2), the approximate solution obtained *via* this computational procedure converges to the "real" (if analytical) solution and also preserve its basic properties (existence, uniqueness and continuous data dependence) as well as its Lyapunov stability. These remarkable features of the procedure recommend it to be used for modeling different other operation and control scenarios in order to evaluate the system behavior under various constraints for equipment damage avoidance.

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